THE EFFECTIVE THERMAL DIFFUSIVITY OF A
BED FLUIDIZED BY VIBRATION

A. I. Tamarin and I. I. Kal'tman   UDC 541.182.3

Measurements are reported for the horizontal component of the effective thermal diffusivity of a bed in a vessel of diameter 20 cm with vibrations of 16-30 Hz and amplitude 2-6 mm.

There have recently been technical applications of fluidization of granular material by vibration [1-3]; one needs to know the effective thermal diffusivity in order to design industrial plant with such beds; some evidence on the effective thermal conductivity of a layer of corundum (particle sizes 0.16 and 0.32 mm) has been given [4, 5] for amplitudes < 1 mm.

We have determined the effective thermal diffusivity of such a bed of sand and also of corundum for vibrations of amplitude more than 1 mm, which are of interest for some applications [1-3].

Heat is transported in space in such a system, as in a normal fluidized bed, by moving eddies of particles (the bulk thermal capacity of the particles is greater by a factor 1000 than that of gas) and this may be described to a first approximation via Fourier's equation:

$$\frac{\partial \theta}{\partial \tau} = a_{ef} \nabla^2. \tag{1}$$

The coefficient a_{ef} for the effective thermal diffusivity characterizes the mixing (diffusion) of the particles, and by analogy with turbulence in a liquid may be expressed as follows [10]:

$$a_{ef} \sim \bar{l} V_p. \tag{2}$$

The energy supply is the basic factor governing the state of the system; the level of this is determined by the frequency and amplitude of the vibrations.

The kinetic energy supplied to the system is transmitted to the particles, and this results in continuous energy dissipation on account of friction between the moving particles and the gas, the walls of the vessel, and the other particles, as is evident from the temperature rise in the vibrating layer [2].

TABLE 1. Averaged Values of a_{ef} (cm^2/sec)

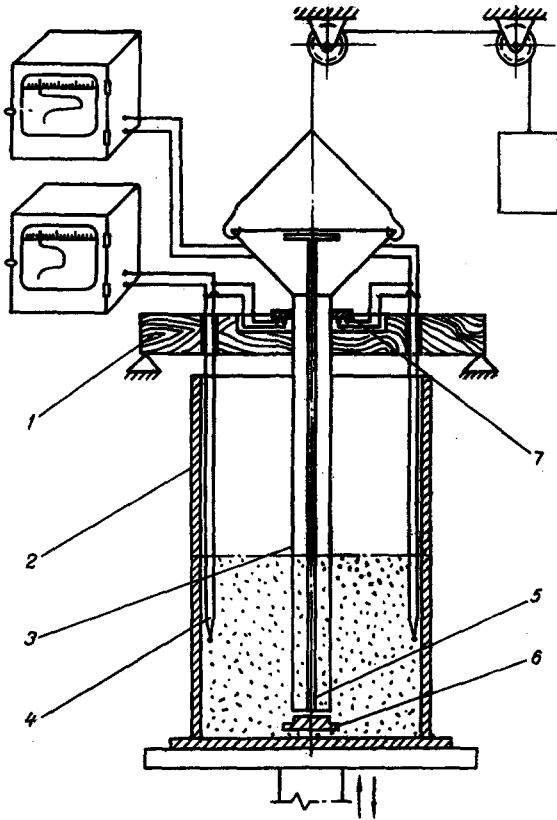| Material	Depth, mm	Frequency, Hz								
		16			20			25		30
		amplitide, mm								
		2	4	6	2	4	6	2	4	2
Sand	100	0,36	1,4	1,6	1,14	1,11	1,8	0,77	1,84	1,25
	200	0,81	1,96	2,1	0,94	1,72	1,94	1,28	1,41	1,77
	300	1,31	1,27	1,23	1,12	0,7	1,24	0,77	1,52	1,08
Corundum	100	2,4	5,8	3,5	1,05	4,3	5,8	1,1	2,1	1,0
	200	1,6	1,78	2,8	2,1	1,5	3,2	1,2	2,9	1,57
	300	0,97	1,42	1,9	0,83	1,76	1,5	0,74	—	0,59

Fig. 1. Scheme of experimental installation.

The higher the kinetic energy input, and consequently the greater the speed of motion, the more energy is dissipated in unit time; the latter is constant in the steady state and is equal to the input energy. Then we have

$$\overline{M} = - k_1 \overline{E}. \tag{3}$$

The energy is supplied to the system by the lower boundary of the layer. It has been found [11] that there is a prominent gas band between a bed fluidized in this way and the bottom of the vessel; as the vessel moves upwards, the height of this boundary decreases and the gas is expelled between the particles; if the rate of gas passage is sufficient (velocity for onset of fluidization), the gas gives up a certain amount of kinetic energy to the particles. When the thickness of this boundary layer becomes zero, the particles contact the vessel; for a certain period the vessel and the particles move together, and then the difference in the accelerations causes them to move away from the bottom of the vessel, which begins to move downwards, so that gas is drawn in again and the gas boundary layer is re-formed.

Energy is brought into the system thus by collision of particles with the vessel and by the gas flow. The power input in the steady state may be defined as

$$\overline{M} = \frac{1}{T} \int_0^T \left[m_1 G \frac{d}{d\tau}\left(\frac{V_{pv}^2}{2}\right) + m_2(1-G)\frac{d}{d\tau}\left(\frac{V_{pg}^2}{2}\right) \right] d\tau, \tag{4}$$

where G is a transmitting function [12], which is 1 when the particles are in contact with the vessel and zero the rest of the time.

The first term in the integral represents the power brought into the layer by collision with the vessel, while the second takes account of the power supplied by the infiltrating gas.

At the moment of collision, the particle velocity is equal to the velocity of the vessel:

$$V_{pv} = A\omega \cos \omega\tau. \tag{5}$$

The particle speed in the steady state is determined by the difference between the gas speed and the lifting speed; the latter is equivalent to V_{cr}, the velocity for fluidization onset, when the concentration is high. We consider the vibrated layer as a quasistationary system to put

$$V_{pg} = A\omega \cos(\omega\tau - \varphi) - V_{cr}, \tag{6}$$

where φ is the phase shift between motions of vessel and particle. From (4)-(6) we get

$$\overline{M} = - k_2 A^2 \omega^3. \tag{7}$$

We express the mean kinetic energy of the particles in terms of the mean velocity:

$$E = k_3 \overline{V}_p^2 \tag{8}$$

and solve (2), (3), (7), and (8) for a_{ef} to get

$$a_{ef} \sim k_4 \sqrt{A^2\omega^3 \, l} = k_5 A\omega^{1.5} \, l. \tag{9}$$

TABLE 2. Results from Statistical Treatment

Material	Depth, mm	Correlation coefficient	Standard deviation, %	Parameters in (11)	
				c	b
Sand	100	82	32	0,85	0,3
	200	71	16	0,34	0,9
	300	5	26	0,03	1,1
Corundum	100	37	88	0,55	1,1
	200	57	30	0,4	1,1
	300	48	43	0,47	0,6

It follows from (9) that the heat transport rate is a function of the square root of the power input, i.e., of $A\omega^{1.5}$; it has been shown by experiment [13, 14] that the rate of vibrational separation and transport for loose materials is determined by the same parameter $A\omega^m$, where $1.37 \le m \le 1.7$.

An interesting point is that $A\omega^{1.6}$ governs the coefficients of heat transfer between the fluidized layer and a surface immersed in it; this has been observed via the turning point on the heat-transfer curves as functions of frequency for various vibration amplitudes [9].

To determine a_{ef} we used the thermal pulse method previously used for fluidized beds and which has been adequately described [6, 7].

Figure 1 shows the apparatus, which consists of a cylindrical vessel 2,200 mm in diameter and 600 mm high (solid bottom), which is set up on the eccentric vibrator to produce vertical oscillations, whose frequency was adjustable from 16 to 30 Hz and whose amplitude was 2 to 6 mm. Above vessel 2 there was a plate 1 with a hole for tube 3 with a funnel; the end of the tube was closed by the bung 6. Rod 5 knocks out the bung. Tube 3 was suspended by a system of blocks and had a counterweight to provide rapid removal from the layer; the upper part of the tube was fitted with the flat metal ring 7, which rested on the wooden sheet 1 when the tube was immersed in the layer and closed the connection between copper plates installed there, thereby shunting the measuring circuit for the two thermocouples 4, which were set up in two diametrically opposite points at half the height of the immobile bed and which were connected to two fast electronic potentiometers (time for full-scale deflection 1 sec).

We used three fractions of sand (114, 200, and 350 μm) and two of corundum (46 and 84 μm); the depths of the layer in the immobile state were 100, 200, and 300 mm.

The vibration amplitude was measured with wedges to 0.2 mm, while the frequency was measured by a tachometer with a coefficient of variation of 3%.

The experiments were conducted as follows; with the system closed, a batch of powder heated to 600–700°C was run in through the tube; then the tube was pushed down into the fluidized bed and the temperature-recording system was switched on. Rod 4 rapidly ejected the cork and then the tube was removed rapidly from the bed. The time for the particles to run out did not exceed 1/10 of the time to attain the maximum temperature, so one can [8] consider the heat input as instantaneous. As the tube leaves the layer, the shunting contact is opened and the moment of onset of recording is marked on the chart paper. The height of the layer of powder in the tube was approximately equal to the depth of the layer in the vessel, which produced conditions for cylindrical (linear) thermal pulsing. Usually, the injected powder constituted ~1% of the total amount present. The material to be heated was taken from the vessel for each successive experiment, so the volume of the bed remained constant. During the measurements we recorded the temperature at the wall, and from the chart we deduced the time to reach the maximum temperature; we calculated a_{ef} from the relationship [6, 7] for a linear source:

$$a_{ef} = R^2/4\tau_m. \tag{10}$$

The maximum relative error in a_{ef} did not exceed 10%.

It was obvious at once on examining the data that the observed a_{ef} had a large spread, which indicates that the fluidized layer was nonuniform; the measured a_{ef} were instantaneous values and local ones, and they characterize only microscopic processes; but the quantities of most practical interest are those relating to macroscopic processes, whose time scales are greater than the relaxation times of nonuniformities

in the system, as well as processes under stationary conditions. In these cases one characterizes the system by the mean effective thermal diffusivity. To average the experimental value we used the results from numerous runs; in a single condition we made six sets of measurements, and in all recorded over 700 experimental points.

There was no definitely established effect from particle size on a_{ef} on account of the spread in the experimental values; only the vibration parameters had ascertainable effects, so the results for each A and n were averaged for all fractions of sand and corundum, and Table 1 gives the resulting averaged a_{ef}.

We then processed the results to give an equation of the type of (9); the parameters of the system in the horizontal direction were unvarying (vessel diameter and position of linear heat source), so we sought a function of $A\omega^{1.5}$; the results were subject to linear regression in a logarithmic coordinate system, with the points represented as the power function

$$a_{ef} = b \left(A\omega^{1.5} \right)^c \text{ [cm}^2\text{/sec]}. \tag{11}$$

Table 2 gives results from the statistical treatment of the data.

The heat-transfer rate is a complicated function of $A\omega^{1.5}$ and layer thickness; if the thickness is small, the heat transfer rate increases rapidly with the input power, but the dependence becomes less pronounced as the layer thickness becomes greater, as is well seen in the tests with sand, where the correlation coefficient falls from 82% to 5% when the height of the layer is increased from 100 to 300 mm. The corundum test had a larger spread in the points, and therefore this relationship was smoothed. In the sand experiments, a 200 mm bed gave higher values of a_{ef} much more often than did a 300 mm one, as Table 1 shows. Table 2 shows that a 200 mm bed (H/D $\approx$ 1) gave minimal spread in the points.

The results for a_{ef} in the sand test agree qualitatively with those of [1], which showed that the maximum coefficient of dynamic activity* in vibrational mixing of sodium bicarbonate and calcined soda at below 30 Hz corresponded to H/D $\approx$ 0.5. This maximum shifts to smaller thicknesses as the frequency is raised, while the absolute value of the coefficient is reduced.

The following is the probable reason for the dependence of the mixing rate on the layer depth; as H increases, the mean speed gradually decreases, but simultaneously there is an increase in the mean scale of the circulation. For a certain H, the increase in circulation scale does not compensate for the fall in particle velocity, and then the mixing rate as regards space will fall.

Interesting results were obtained in [5] for corundum with mean particle sizes of 0.16 to 0.32 mm in a vibrating vessel of diameter 154 mm; the heat-transfer rate was determined from the temperature distribution, and the tests were done at high frequencies (69 Hz) and low amplitudes (0.18-0.33 mm). The resulting a_{ef} were about 0.2-0.4 cm^2/sec, which were virtually the same as in our experiments for similar values of $A\omega^{1.5}$. An interesting point about [5] is that the results were presented in coordinates of effective thermal conductivity against vibration power.

These measurements show that the a_{ef} for a fluidized bed is 1-3 cm^2/sec, i.e., close to the a_{ef} in an ordinary fluidized bed [7]. The a_{ef} of a vibrating bed is a function of the input power, i.e., of $A\omega^{1.5}$; it increases with this parameter specially rapidly when the bed is shallow. The effects of this factor gradually become less as the bed becomes deeper, and from a depth of about 200 mm onwards there is a fall in the a_{ef}, which implies a reduced rate of mixing in the bed. This indicates considerable dispersal of the kinetic energy as the depth increases.

NOTATION

A, n	amplitude and frequency of vessel fluctuations;
a_{ef}	effective thermal diffusivity of vibrofluidized bed;
$\overline{M}$	mean power supplied within fluctuation period;
$\overline{E}$	mean kinetic energy of moving particles for a cycle;
k	proportionality factor;
V_{pv}	velocity of particles colliding with vessel at collision moment;

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* A quantity inversely proportional to the time of mixing of the components. Experiments [1] were conducted in vessels of cross section 240 × 240 mm.

V_{pg}	particle velocity;
V_{cr}	velocity of fluidization onset;
m_1, m_2	mass of colliding particles and mass carried away by gas;
$\bar{l}, V_p$	mean scale of circulation and mean velocity of material;
ω	circular frequency;
τ	time;
τ_m	time of maximum temperature onset;
H	height of filling;
D	diameter of vessel;
T	fluctuation period;
d_{av}	mean diameter of particles;
R	radius of vessel.

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